Equity Market Impact Models

Mathematics at the interface between business and research

Stifterverband für die Deutsche Wissenschaft

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Outline

Transaction costs and market impact

- Three key components of the investment process
  1. alpha
  2. risk
  3. costs
- alpha and risk heavily studied by academics and practitioners
- cost aspect often assumed away in academic research

Deutsche Bank conducts extensive fundamental and practical research on transaction cost analysis in close collaboration with QP-lab

Transaction costs and market impact

Trade costs naturally divide into two parts

1. Direct costs such as commissions, custody fees, taxes, and infra-structure costs
   - primarily determined by quantity of trading
   - easy to measure
2. Indirect costs such as market impact and opportunity costs
   - primarily determined by the trading strategy at micro-level
   - hard to measure

this presentation is about market impact: why is it important, how can it be measured, how does it evolve over time?

Illustration I : MI & portfolio manager's alpha

Execution costs are a key determinant of investment performance.

- It is now widely recognised that they can substantially reduce an investment strategy notional performance.
- The average cost of a US large cap trade trade over the last 5 years is 23 bps (ITG Global Trading Cost Review - Aug 2008).
  - Costs = Commissions (9 bps) + Market impact (14 bps)
- Assuming an average annual turnover of 100%, transaction costs reduce the performance of a US large cap fund by 45 bps p.a.
- US large cap funds underperformed the S&P 500 by 40 bps p.a. over 5 years ending June 08 (SPIVA US Scorecard Mid Year 08).

Illustration I : MI & portfolio manager's alpha

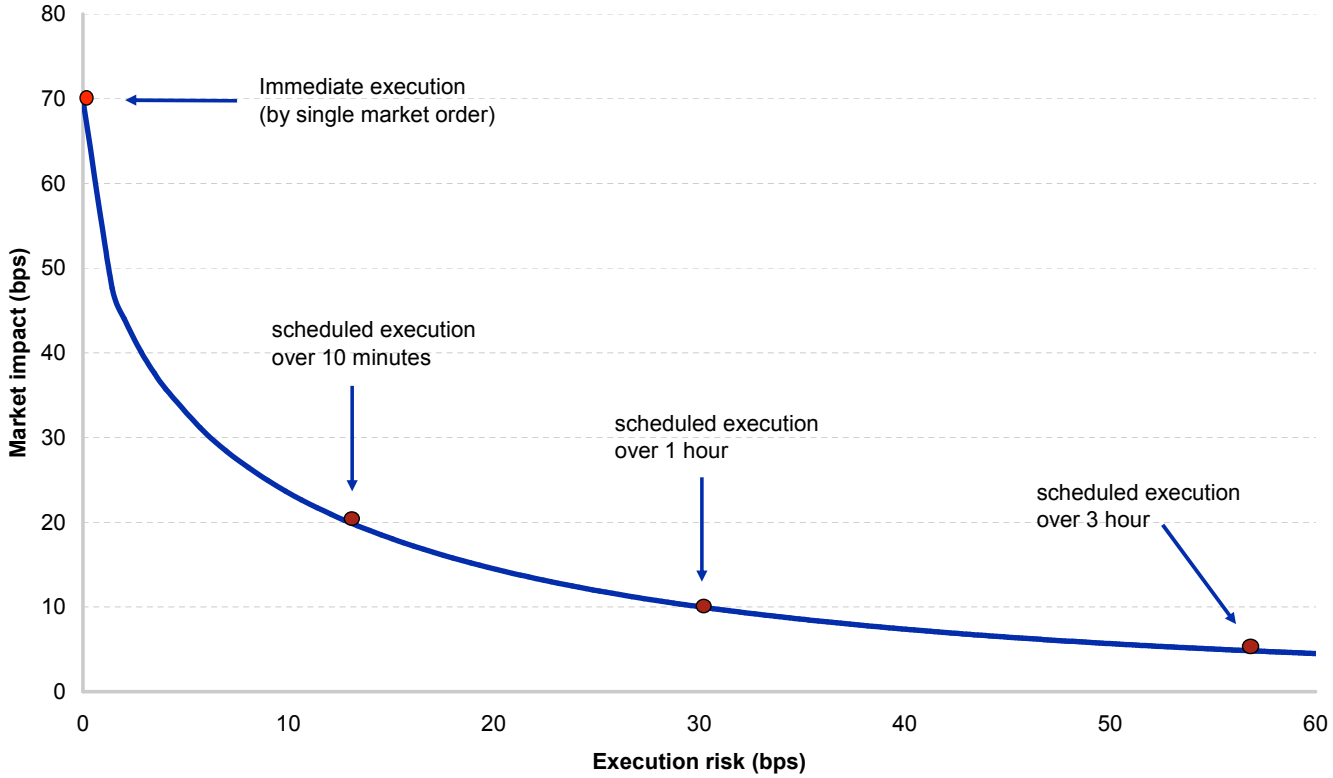
Coppejans & Madhavan (2007) examine how transaction costs impact ex-ante Information Ratios.

- Transaction costs can decrease the IR substantially.
  - For a typical stylised fund, assuming 40 bps transaction costs and 200% turnover, Coppejans & Madhavan (2007) show that IR is halved when transaction costs are taken into account.
- IR is also determined by the correlation between predicted and realised costs.
  - This introduces the concept of cost skill analogous to the concept of skill (Grinold 1989).

Improving the quality of transaction cost forecasts improves expected performance.

Illustration II : MI & algorithmic trading strategies

Accurate forecasts of execution costs are critical in determining an optimal execution strategy



- Under-estimate costs $\longrightarrow$ trade too fast with higher impact
- Over-estimate costs $\longrightarrow$ trade too slow with more risk

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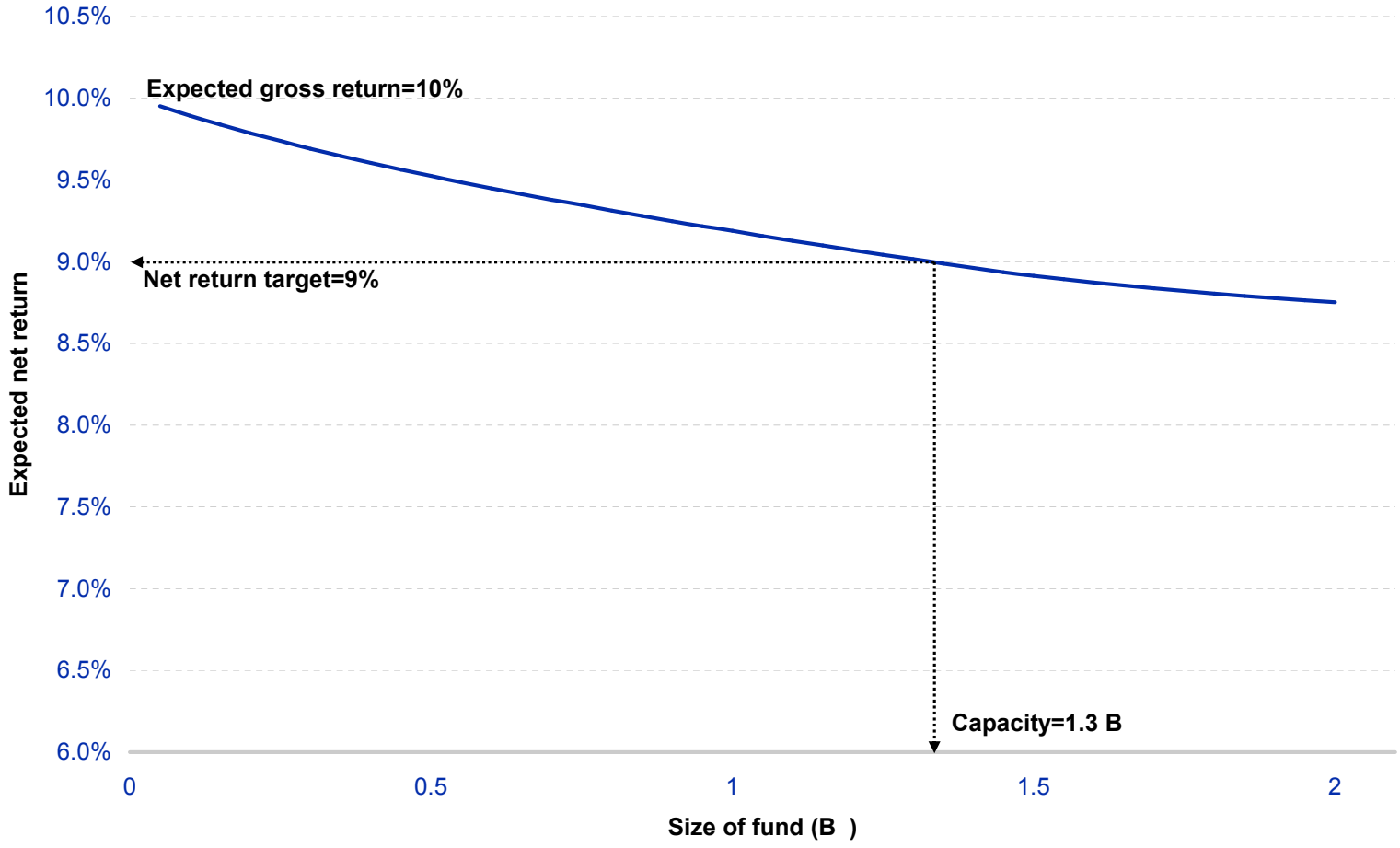
Illustration III : MI & strategy capacity

Execution costs limit investment capacity.

- Larger trade sizes result in higher market impact costs. Beyond a certain size threshold, the net alpha of a trade can become too low.
- DB has used its market impact model to determine the capacity of its funds in a way that protects their alpha.

Illustration III : MI & strategy capacity



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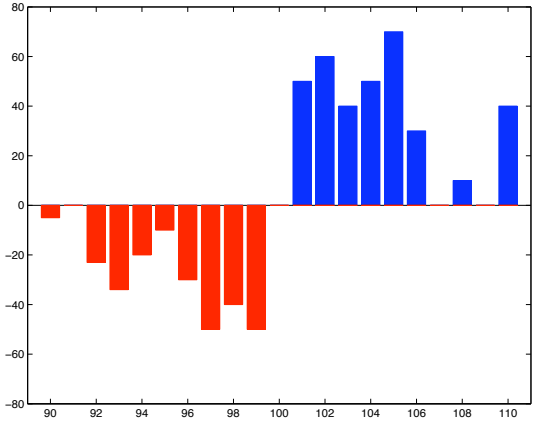
Outline

Decomposition of market impact

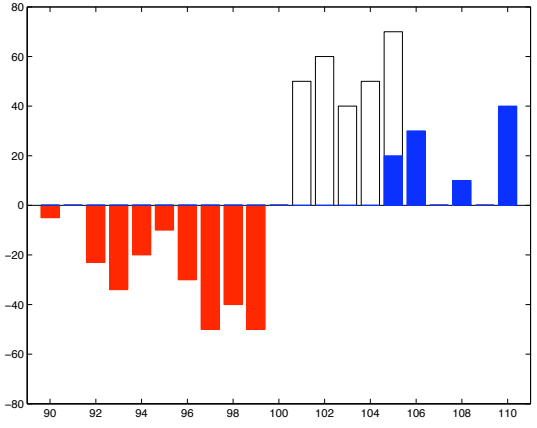
- Market impact pertains to the costs incurred by extracting liquidity from the market in order to acquire or dispose of a position
- In the implementation-shortfall framework of Perold (1988, JPM), MI it is defined as the difference between pre-trade or paper price and the realized execution price
  - liquidity characteristics (such as spread, depth, resiliency, etc) of the stock as well as your own trading behavior determine MI
- MI can be decomposed into three distinct components
  1. instantaneous impact
  2. temporary impact
  3. permanent impact

Illustration

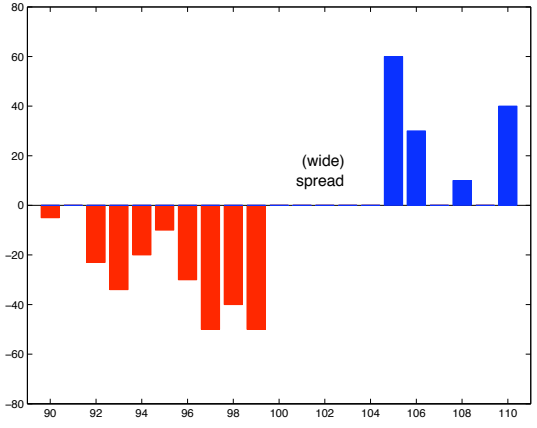
(i) initial state



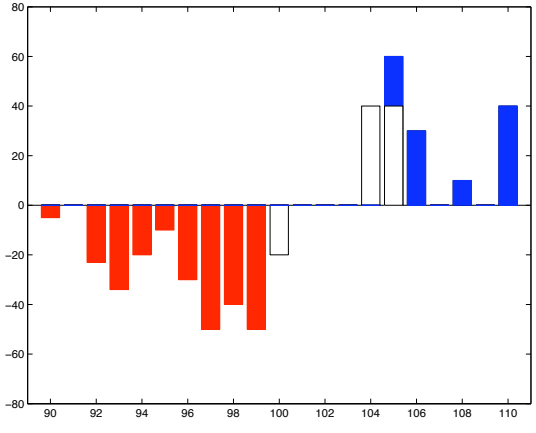
(ii) liquidity is taken



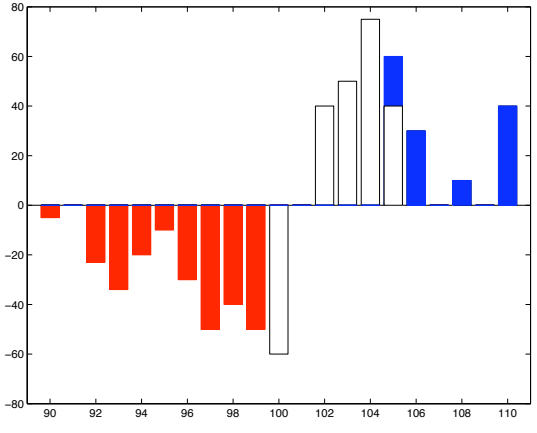
(iii) temporary impact



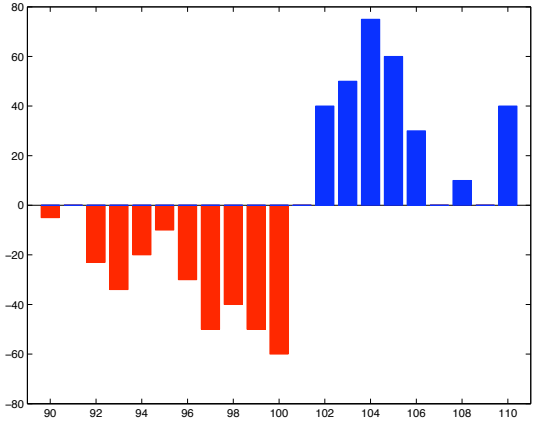
(iv) liquidity returns



(v) liquidity returns



(vi) final state



Instantaneous impact or spread crossing costs

- Demsetz (1968, QJE) justifies the existence of the bid/ask spread as a compensation for providing liquidity to those that seek it
  - limit orders post liquidity and get paid the spread upon execution
  - market orders take liquidity by crossing the spread
- The frequency with which we need to cross the spread to fulfill our order contributes to MI
  - the more aggressive the strategy, the more often the spread is crossed
  - the more "unskilled" the trader is, the more often the spread is crossed
- Key variables: spread, trading rate

Temporary impact

- By taking liquidity out of the order book, we affect prices by (temporarily) distorting the demand/supply equilibrium
  - temporary impact (by definition) dissipates over time
  - the speed depends on the "resiliency" of the market, i.e. its ability to absorb liquidity demand
- Temporary impact affects the execution quality of subsequent orders
- The choice of trading schedule is crucial in managing the accumulated temporary impact
- Key variables: trading rate, volatility, resilience

Permanent impact

- The Kyle (1982, E) model formalizes intuition that information is revealed through trading
  - informed traders hide behind the flow of "noise" traders
  - market maker infers information content of trades from order flow (the larger the trade, the more information is revealed)
  - impact determined by % order size
- permanent impact is linear in size and symmetric between buy and sell orders (by arbitrage argument of Huberman and Stanzl 2004 E, Gatheral 2008)
- Permanent impact does not decay (by definition) and thus affects subsequent executions *and* inventory
- Key variables: relative order size

Components of MI

	instantaneous ("skill")	temporary ("liquidity")	permanent ("information")
Panel A: Determinants			
spread	✓		
resilience		✓	
order size			✓
trade rate	✓	✓	
Panel B: Price impact			
execution	✓	✓	✓
inventory			✓

Commercially available MI through Bloomberg

- Bloomberg market impact model

$$MI = \underbrace{\frac{1}{2}\frac{S}{P}}_{\text{(spread costs)}} + \underbrace{\sqrt{\frac{\sigma^2/3}{250}}\sqrt{\frac{V}{0.3EDV}}}_{\text{(perm. impact)}}$$

  - MI increases with (i) spread, (ii) volatility, (iii) relative order size

- JP Morgan market impact model

$$MI = \underbrace{\frac{5}{100}I}_{\text{(perm. impact)}} + \underbrace{1.4\frac{95}{100}\frac{V}{EPV}I}_{\text{(temp. impact)}} \quad \text{where} \quad I = 0.187\sqrt{\frac{V}{EDV}}\sigma^2$$

  - MI increases with (i) variance, (ii) relative order size, (iii) trading rate

(spread "S", price "P", volatility "σ", order size "V", expected daily volume "EDV", expected period volume "EPV")

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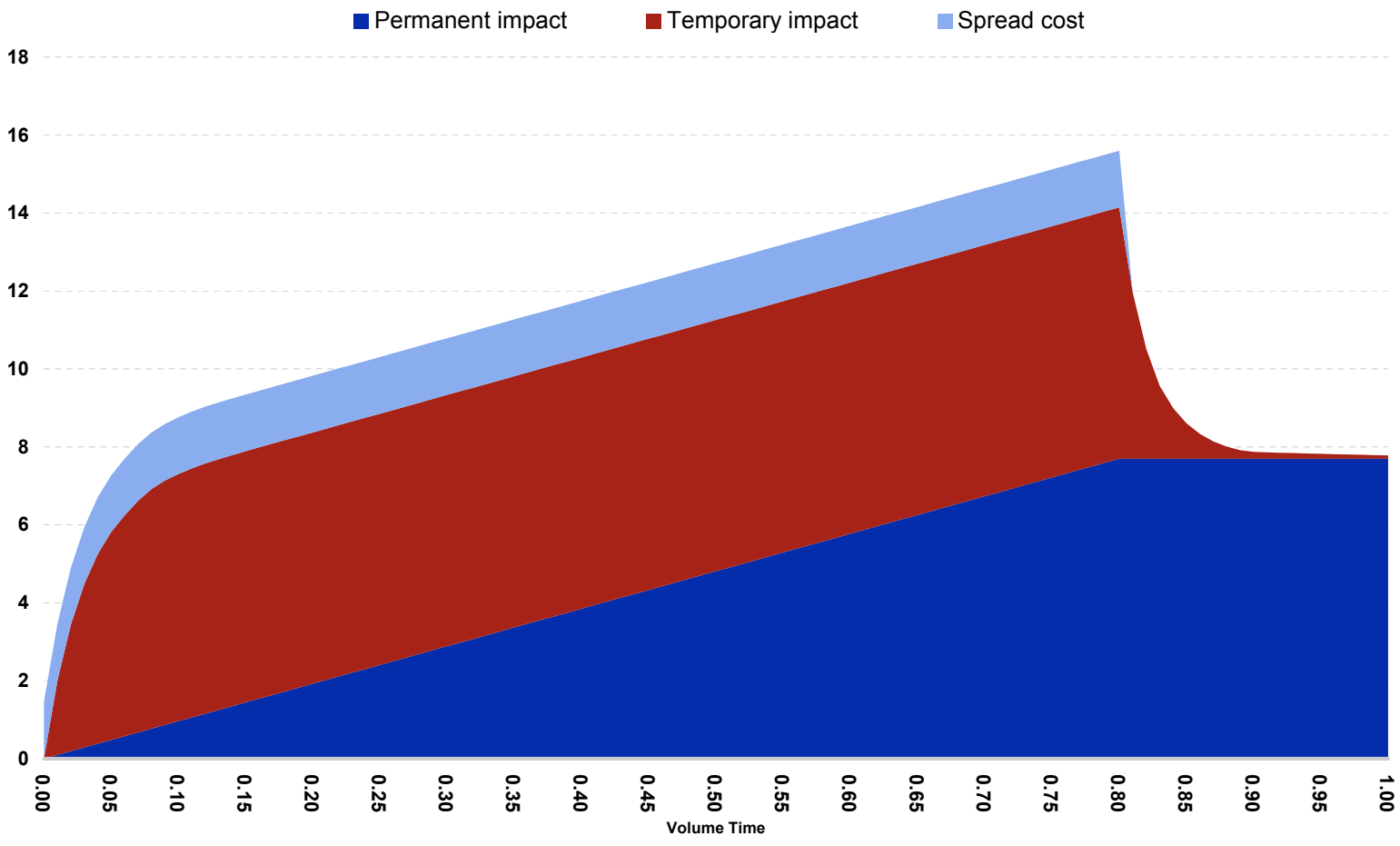
- DB developed a proprietary market impact model

$$MI = \underbrace{g\left(\frac{V}{EDV}\right)}_{\text{(perm. impact)}} + \underbrace{\frac{\widehat{V}}{EPV} f(t,\sigma)}_{\text{(temp. impact)}} + \underbrace{\frac{\widehat{V}}{EPV} h\left(\frac{S}{P}\right)}_{\text{(spread costs)}}$$

  - MI increases with (i) relative order size, (ii) volatility, (iii) trading rate, (iv) spread
  - allows for full decomposition of MI into instantaneous, temporary, and permanent impact
  - allows for MI trajectories over the life of the trade and beyond
  - can handle non-constant trading rate trajectories (e.g. implementation shortfall strategy)

Decomposition of MI using DB model



(VWAP order with size of 10% of EDV, traded over 4/5 of trading day in volume time)

Outline

Investigating Model Performance

- model calibration using non-linear regression

$$p_{\text{exec}} - p_{\text{pre}} = f(V, EDV, EPV, \sigma, S) + \varepsilon$$

- use 166,275 DB handled orders from Jan '07 - Mar '08

	5%	25%	median	75%	95%
realized impact (in bps)	-30.9	-1.80	7.81	26.48	91.55
trade size (as % ADV)	0.05	0.23	0.78	2.55	9.84
trading rate (as % V/PV)	6.50	13.72	20.56	28.53	44.13
trade period (in mins)	1.31	5.16	14.14	38.68	177.12

- The orders cover a wide spectrum of trading rates
- calibration specific to DB execution process (i.e. "skill")

Investigation of Market Impact Model Performance

$$\text{JPM} = \frac{5}{100} I + 1.4 \frac{95}{100} \frac{V}{EPV} I \qquad R^2 << 0$$

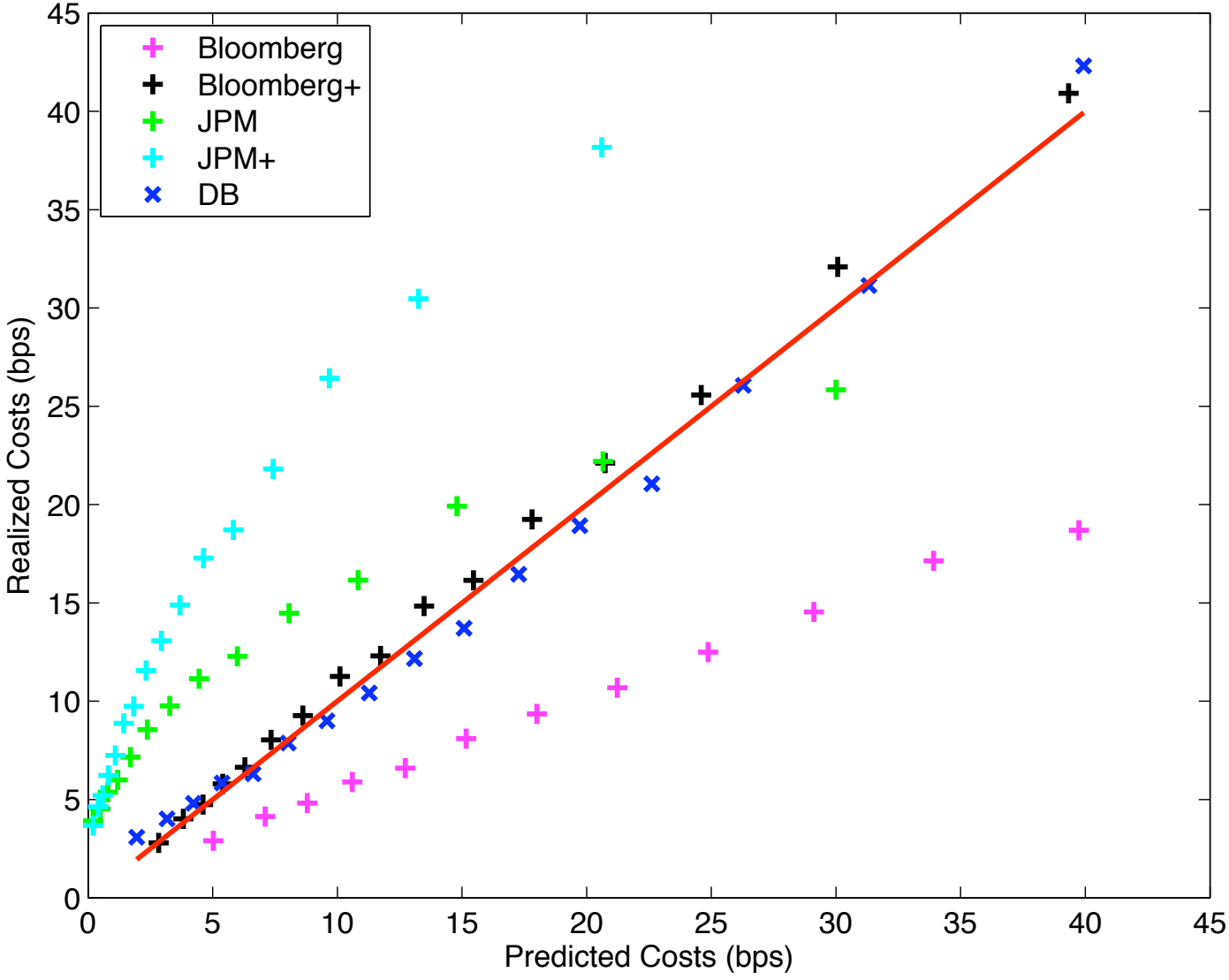
$$\text{JPM+} = \underset{(26.2)}{0.148} I + \underset{(13.2)}{0.240} \frac{\widehat{V}}{EPV} I \qquad R^2 = -1.01\%$$

$$BB = \frac{1}{2} \frac{S}{P} + \sqrt{\frac{\sigma^2/3}{250}} \sqrt{\frac{V}{0.3EDV}} \qquad R^2 = -31.8\%$$

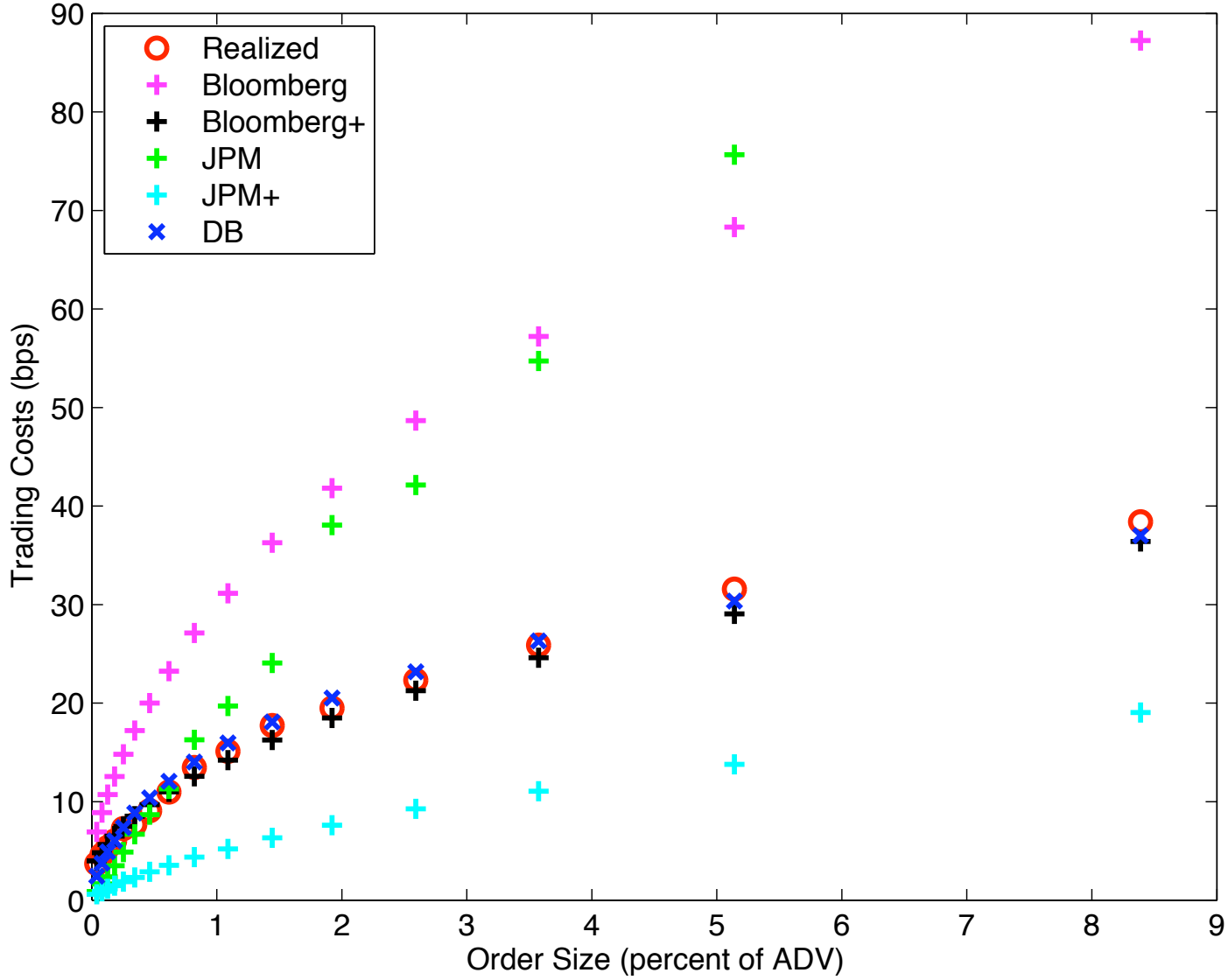
$$BB+ = \underset{(47.2)}{0.433} \frac{S}{P} + \underset{(70.6)}{0.335} \sqrt{\frac{\sigma^2/3}{250}} \sqrt{\frac{V}{0.3EDV}} \qquad R^2 = +8.62\%$$

$$\text{DB} = g\left(\frac{V}{EDV}\right) + \frac{\widehat{V}}{EPV}\left[f(t,\sigma) + h\left(\frac{S}{P}\right)\right] \qquad R^2 = +9.61\%$$

Conditional Performance : realised vs predicted costs



Conditional Performance : size



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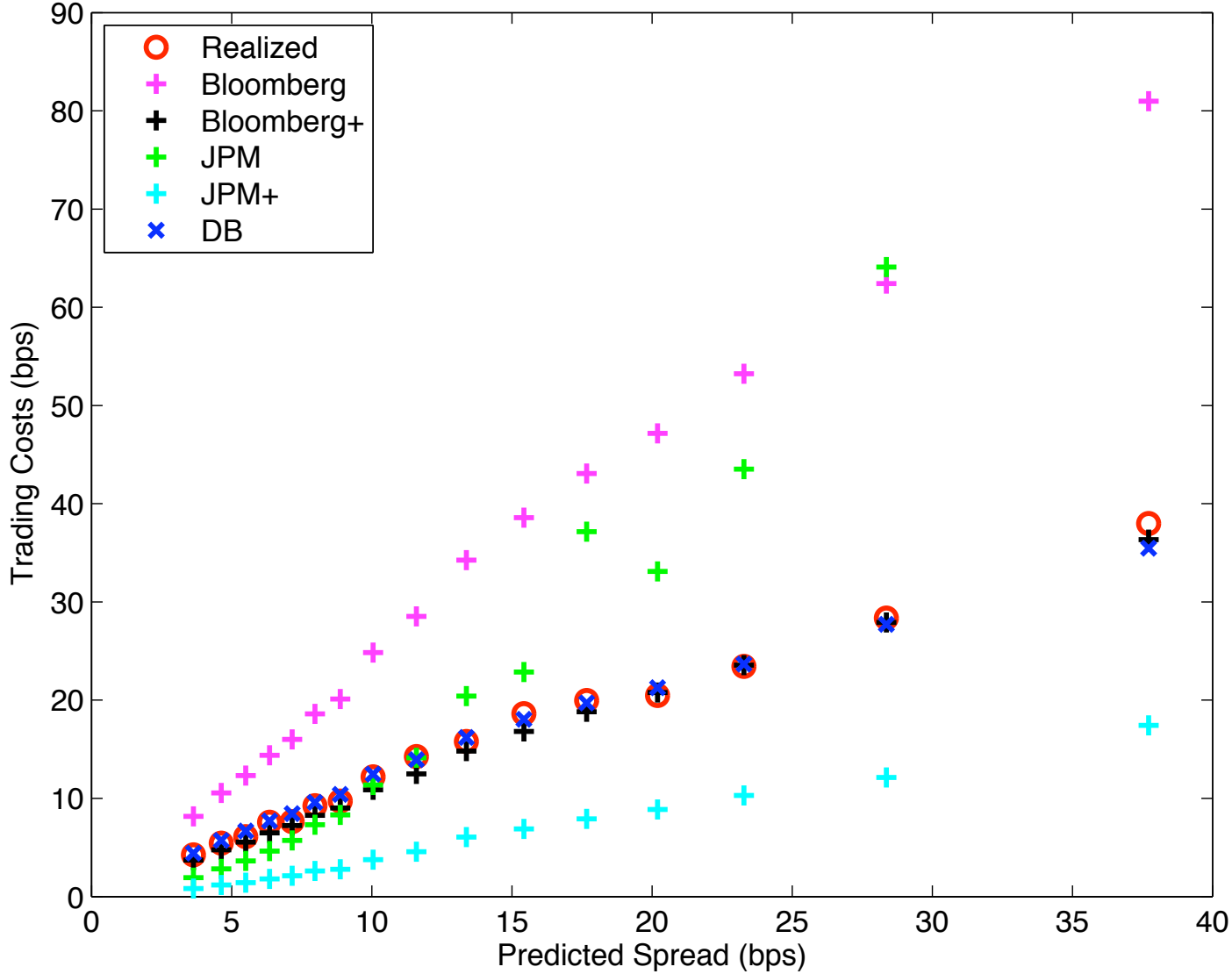
Conditional Performance : spread



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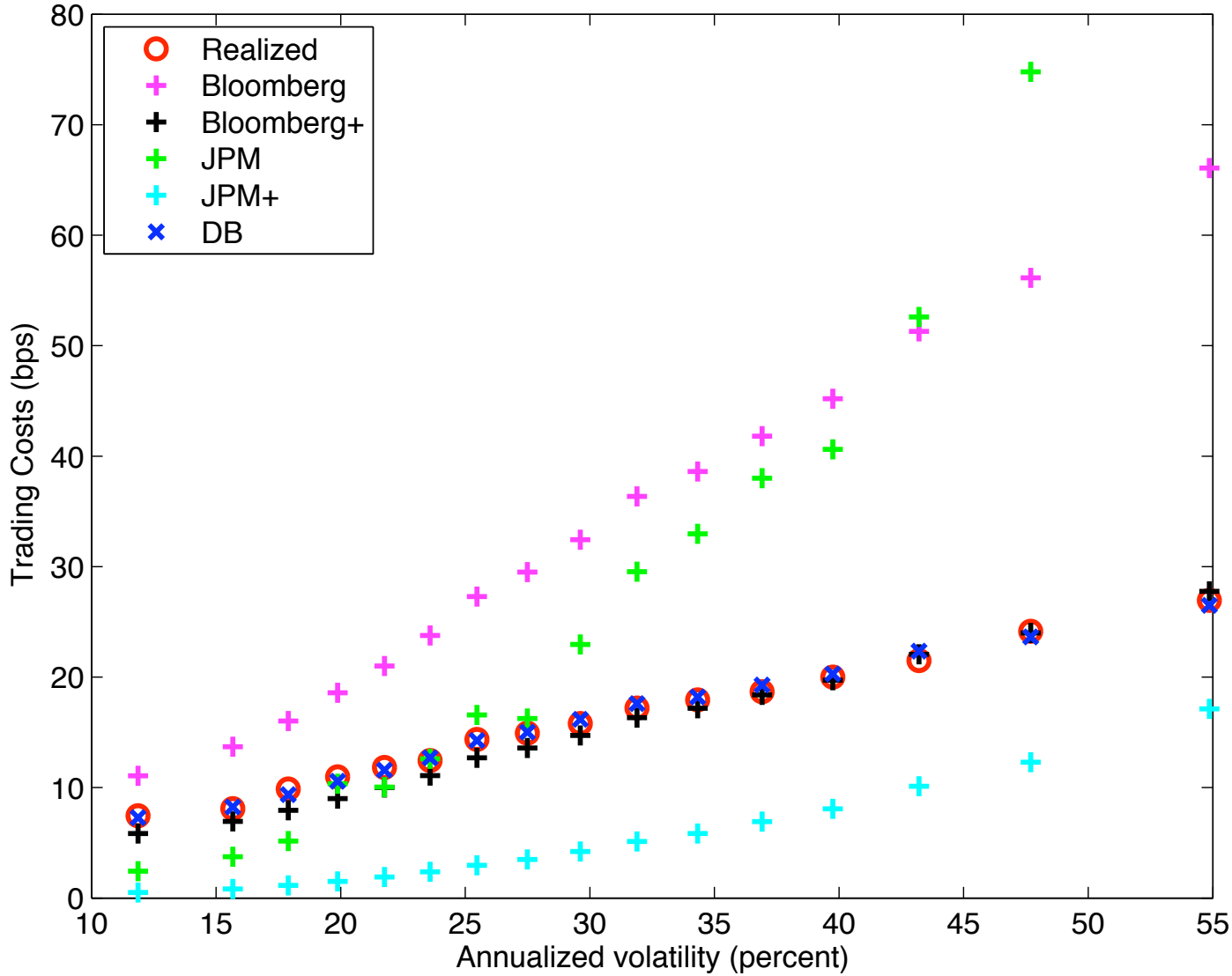
Conditional Performance : volatility



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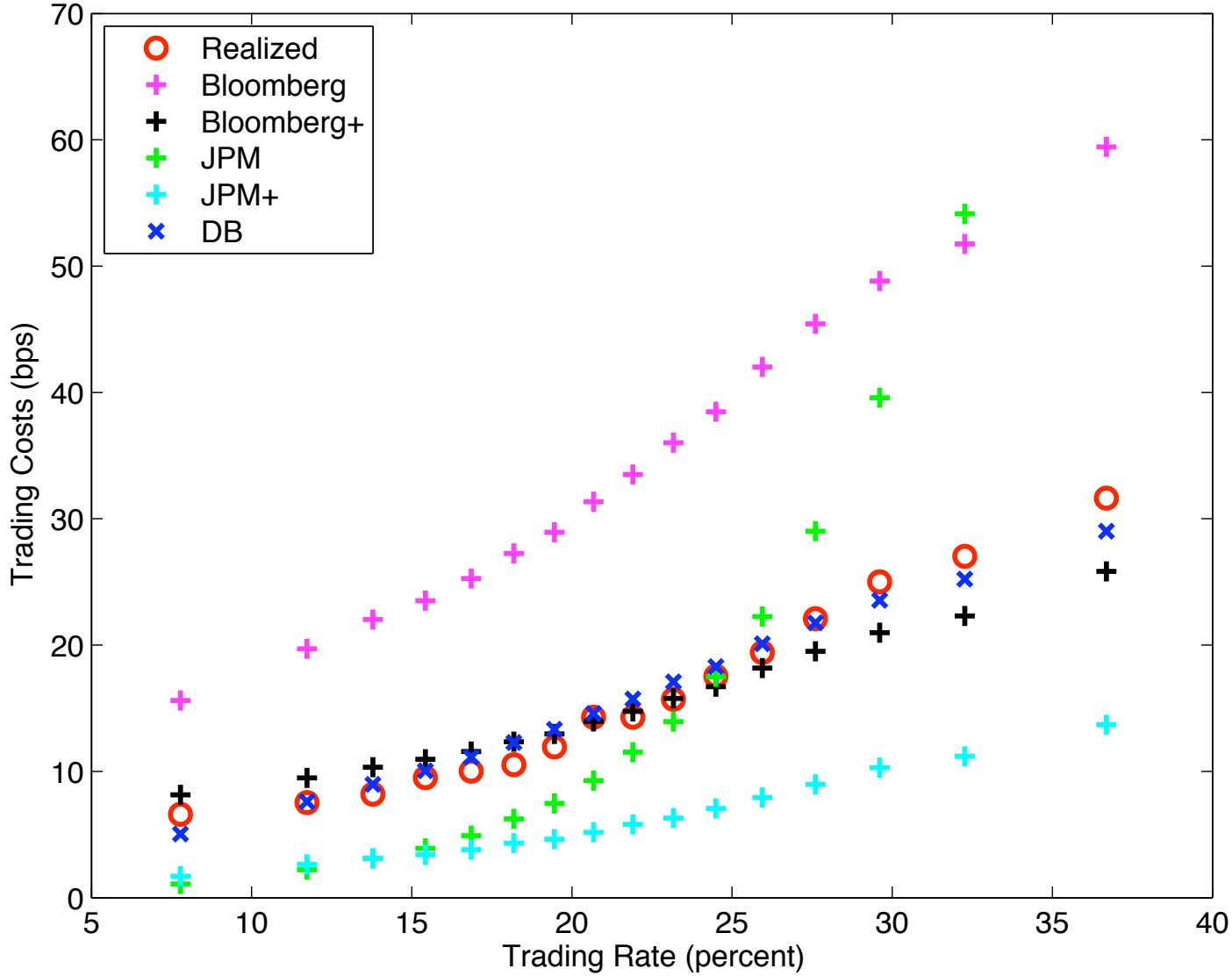
Conditional Performance : trading rate



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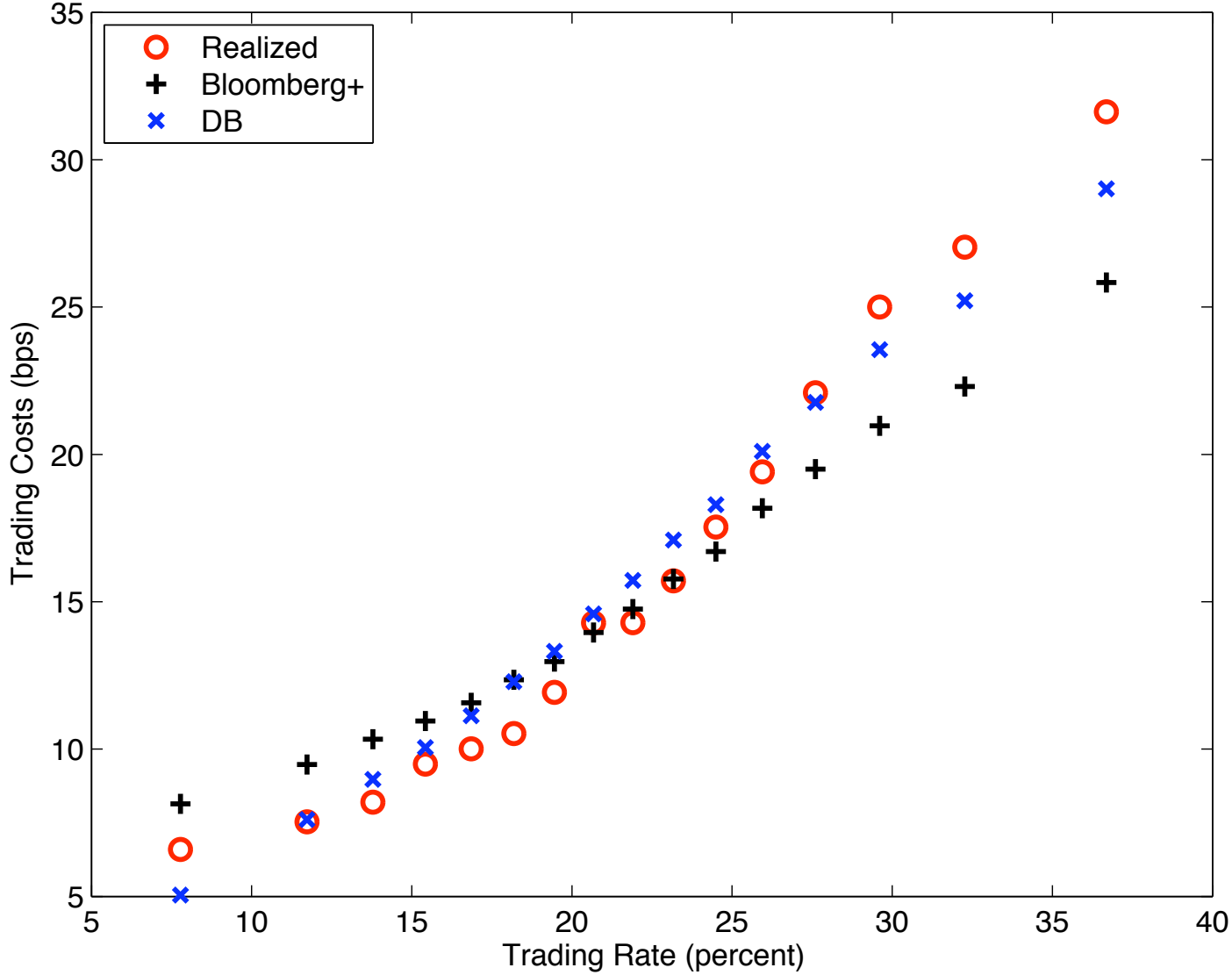
Conditional Performance : trading rate



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Key Observations on Market Impact Modeling

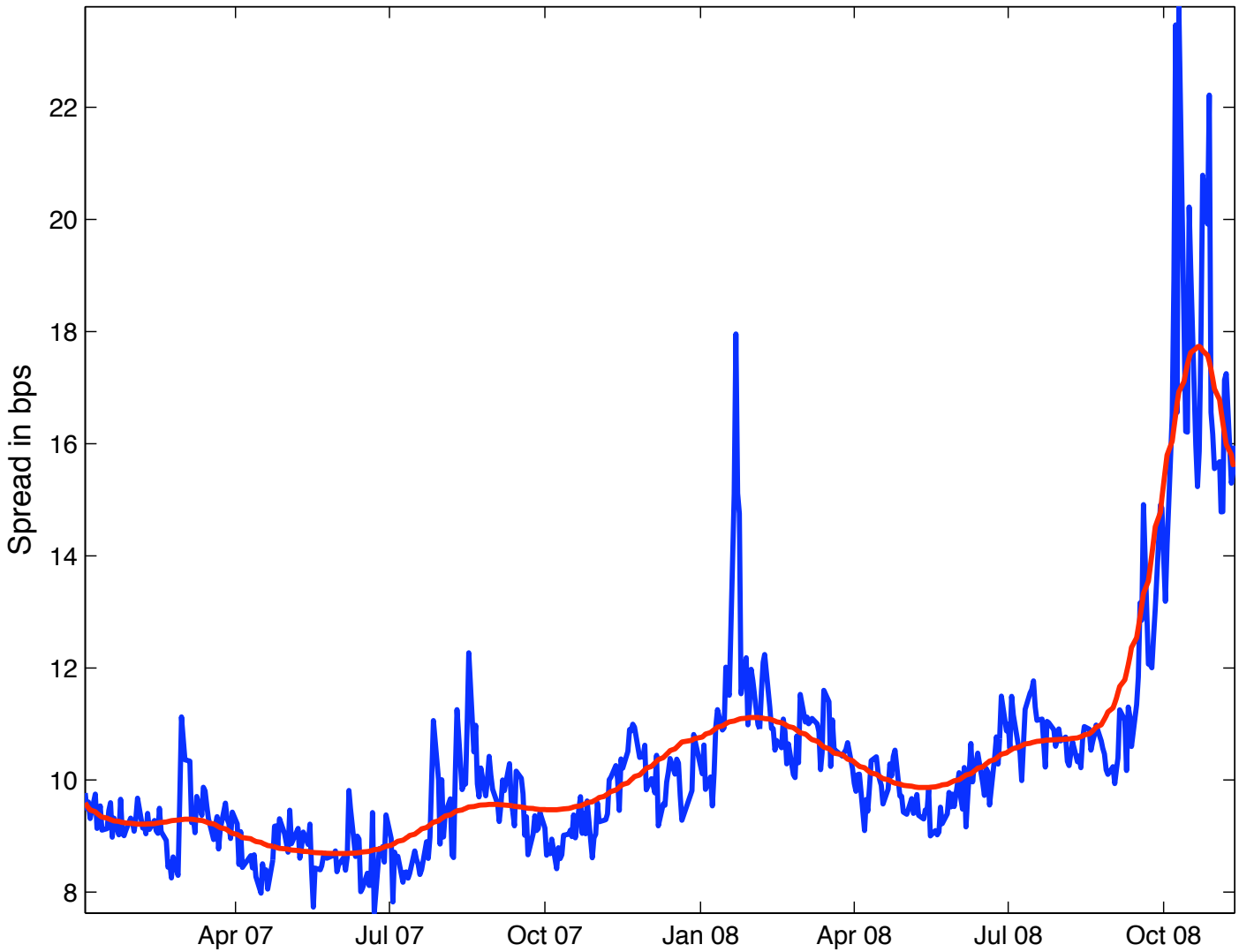
- BB and JPM models fit very poorly to DB executions
- JPM functional form inappropriate (it scales with σ^2)
- BB functional form quite good, but lacks dependence on trading rate
- DB model has superior fit to the data, both unconditionally and conditionally

Outline

MI in a rapidly changing market environment

- Aug - Sep 2007 : start of credit crunch
  - BNP Paribas suspended operations of three of its funds
  - Northern Rock nationalized
- Jan 2008
  - Société Générale incurs trading losses of about 5bn euros (Jerome Kerviel)
- Sep 2008 - date : global recession fears
  - wave of bankruptcies, forced mergers, and restructuring in financial services industry
    (e.g. LEH bankruptcy, BoA buys ML, AIG bailed out by FED)
  - Short-selling of financial stocks banned
  - FED rescue package of $700bn

Key determinants of MI : spreads



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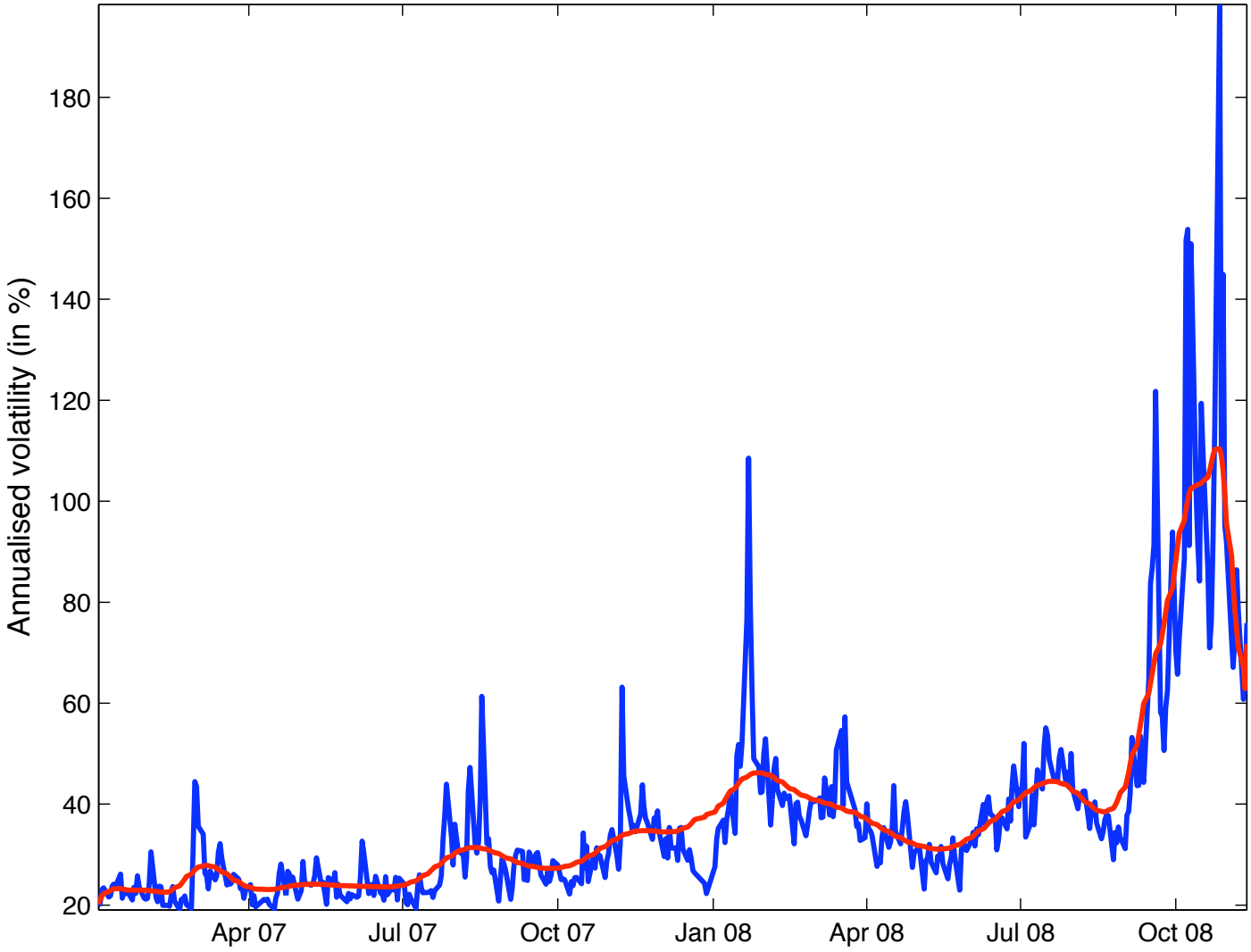
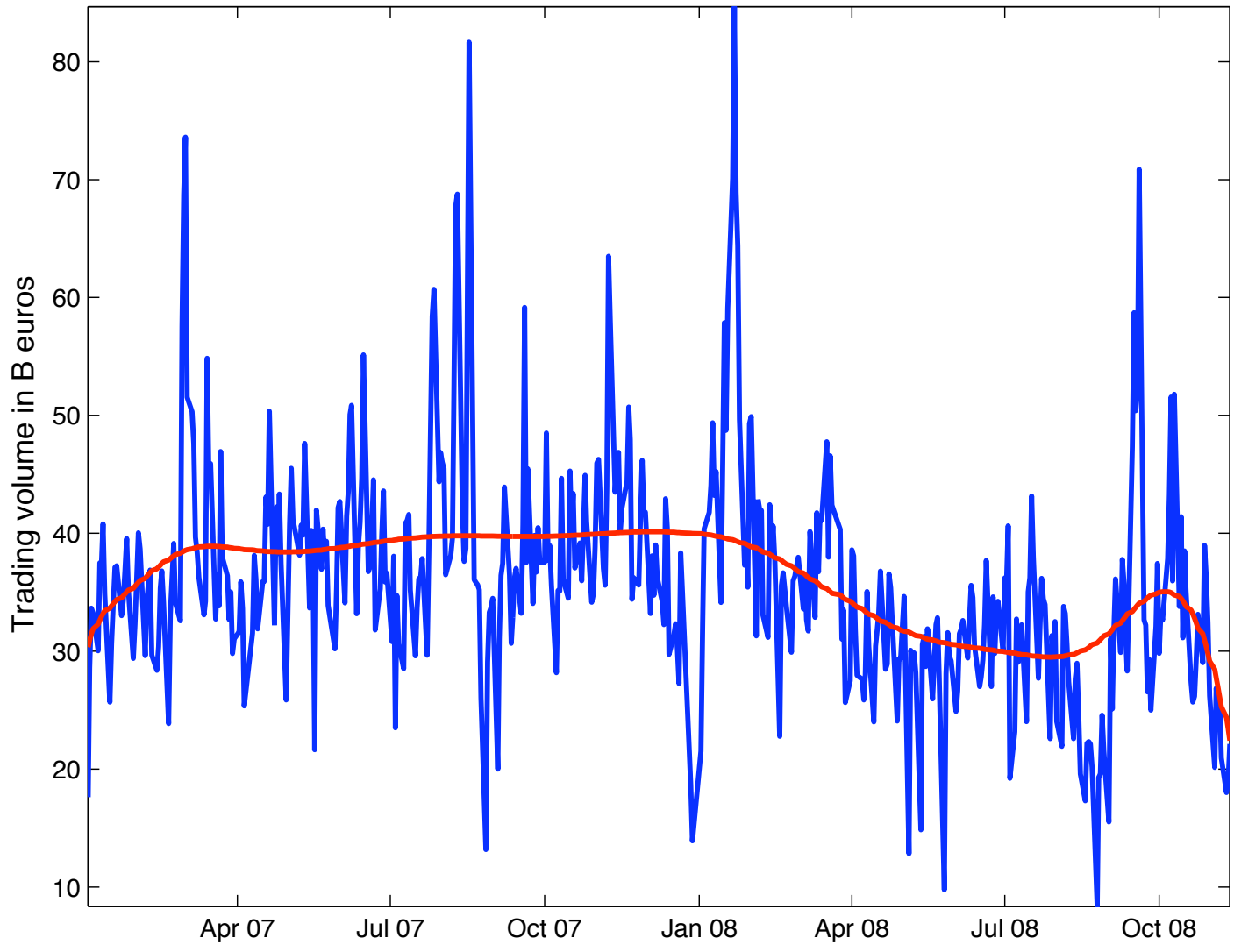
(spreads for volume weighted STOXX600 universe)

Key determinants of MI : volatility



(realized volatility for volume weighted STOXX600 universe)

Key determinants of MI : trading volume



(aggregate trading volume for STOXX600 universe)

Illustrative example

	Apr 07	Aug 07	Jan 08	May 08	Oct 08
σ	20%	50%	90%	30%	150%
spread bps	9	12	15	10	20
EDV	100	100	150	75	60
	VWAP of size 5 from 08:00 - 16:30				
Permanent	3.8	3.8	2.6	5.1	6.4
Temporary	5.5	10.4	11.9	8.8	30.3
Spread	1.0	1.3	1.2	1.3	2.9
Total	10.3	15.6	15.7	15.1	39.5
JPM	1.9	12.2	26.1	6.0	195.3
BB	34.3	80.5	117.0	56.6	298.7

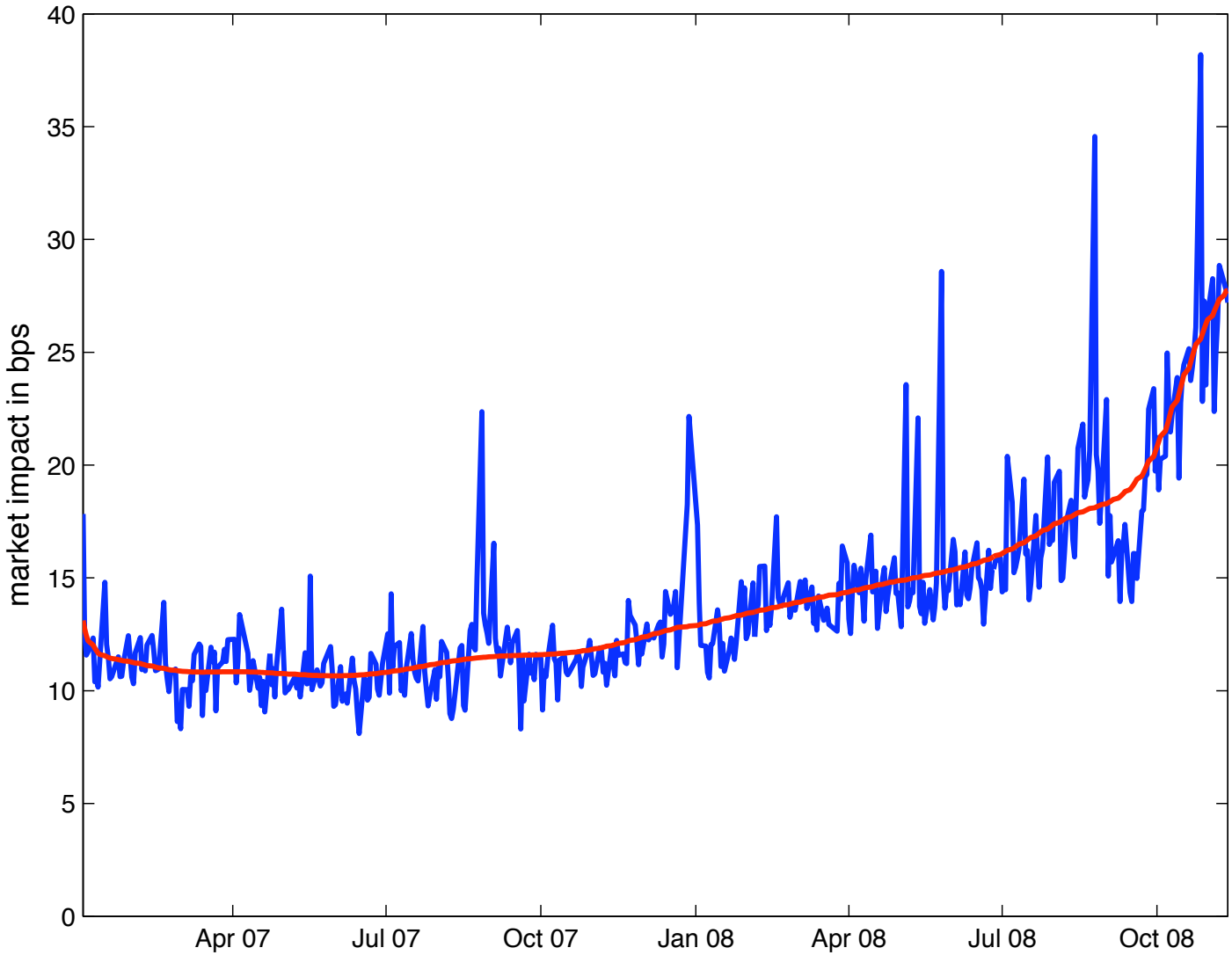
- In terms of market impact, Aug 07 is comparable to Jan 08 and May 08
- Oct 08 is the worst trading environment with market impact more than double

Illustrative example

	Apr 07	Aug 07	Jan 08	May 08	Oct 08
σ	20%	50%	90%	30%	150%
spread bps	9	12	15	10	20
EDV	100	100	150	75	60
	VWAP of size 5 from 10:00 - 13:00				
Permanent	3.8	3.8	2.6	5.1	6.4
Temporary	9.7	18.3	22.4	14.7	49.3
Spread	1.7	2.3	2.3	2.2	4.7
Total	15.3	24.4	27.2	22.0	60.4
JPM	4.0	24.9	48.6	13.1	442.1
BB	34.3	80.5	117.0	56.6	298.7

- Higher trading rate increases temporary impact and spread costs

DB market impact estimate



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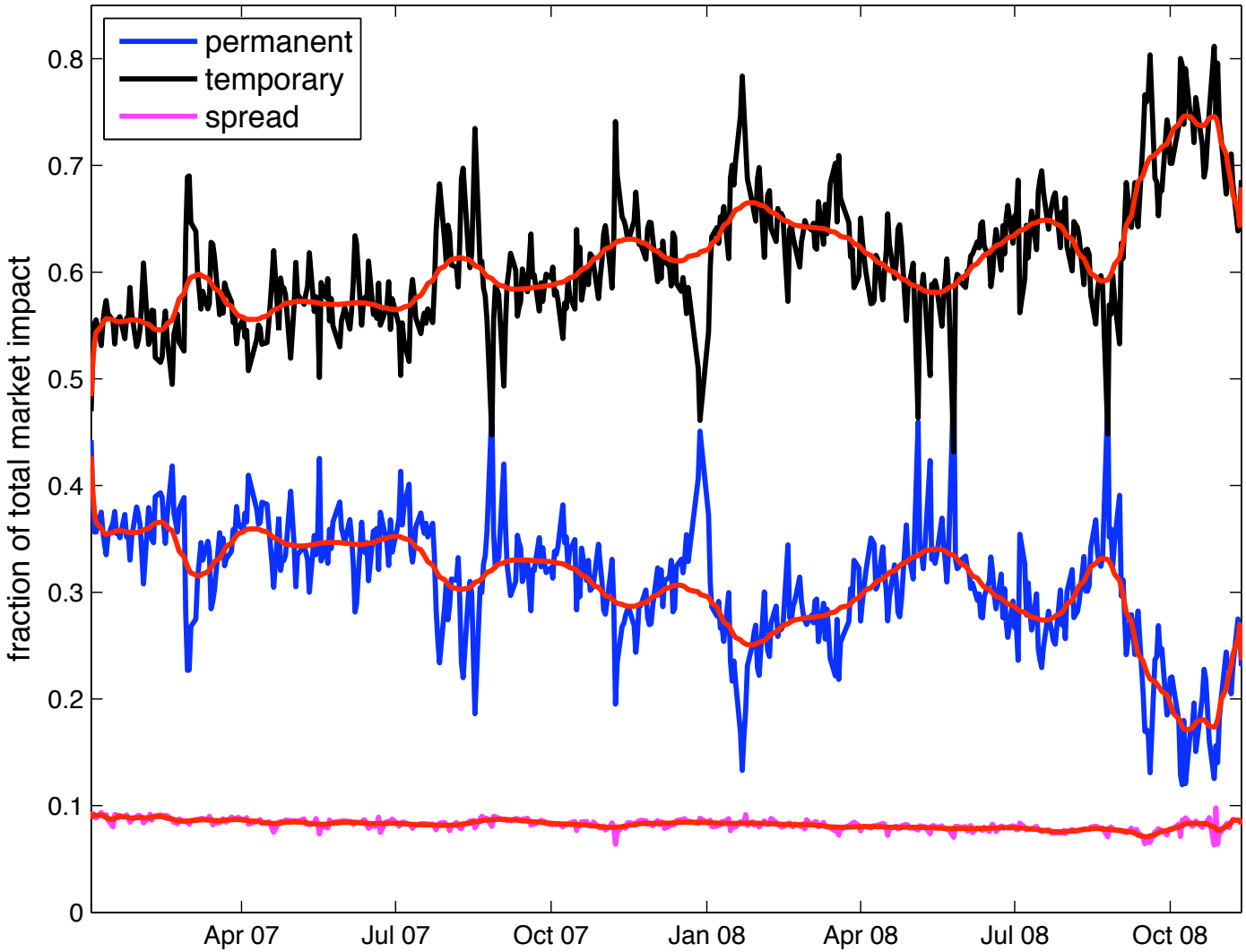
(VWAP of size 5 traded over full day)

Decomposition of total market impact



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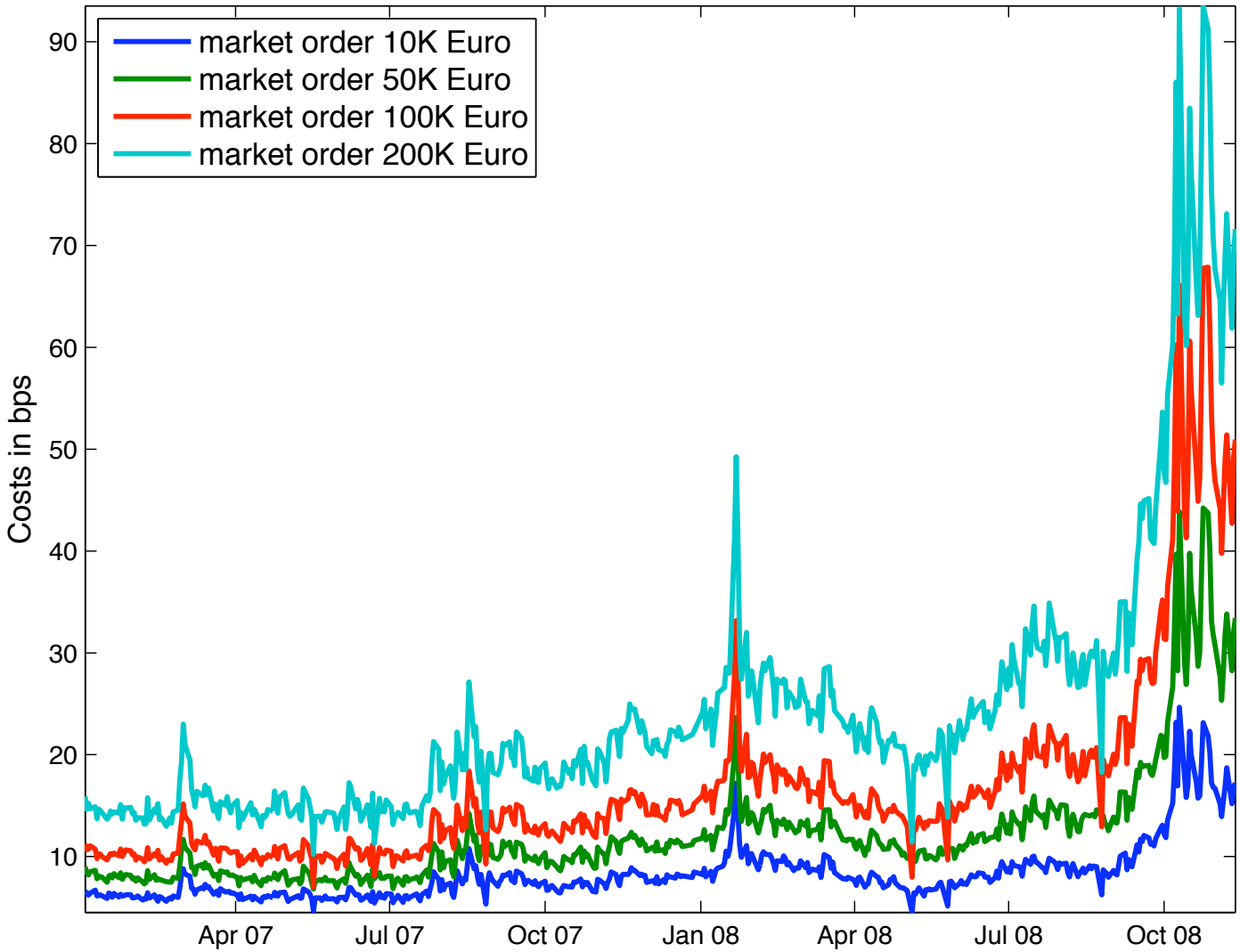
(VWAP of size 5 traded over full day)

Key observations on market impact evolution

- MI has steadily grown over the past year
  - for a typical order in a typical stock, MI has roughly doubled
- contribution of temporary impact has been rising
  - volumes gradually fall, this affects both temporary and permanent impact in same direction
  - volatility substantially up, this only affects temporary impact

Cost of market order of fixed notional



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(median over STOXX600 universe : model-independent)

Outline

Current research topics

QP1, in close cooperation with QPLab, currently conducts research in

- market impact of (i) limit orders, (ii) large multi-day executions
- large dimensional covariance forecasting for ultra-short horizons
- statistical factor models
- mathematical and statistical modeling of limit order books and hidden liquidity
- Bayesian model averaging and forecasting
- ...

Thank you very much for your attention !